需要再次熟悉的内容

1. 二元函数的泰勒公式
   $f(x_0+\Delta x,y_0+\Delta y)=f(x_0,y_0)+\sum_{k=1}^{n}\frac{1}{k!}(\Delta x\frac{\partial}{\partial x}+\Delta y\frac{\partial}{\partial y})^k f(x_0,y_0)+R_n$
    1. 拉格朗日余项$R_n=\frac{1}{(n+1)!}(\Delta x\frac{\partial}{\partial x}+\Delta y\frac{\partial}{\partial y})^{n+1}f(x_0+\Delta x,y_0+\Delta y)$
    2. 佩亚诺余项$R_n=o(\rho^n),\rho=\sqrt{(\Delta x)^2+(\Delta y)^2}\to 0$
    3. 约定$(\Delta x\frac{\partial}{\partial x}+\Delta y\frac{\partial}{\partial y})^k f=\sum_{i=0}^{k}C_k^i(\Delta x)^{k-i}(\Delta y)^i\frac{\partial^k f}{\partial x^{k-i}\partial y^i}$

题

1. （链式法则）设$z=z(x,y)$满足$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=z^2$，设$u=x,v=\frac{1}{y}-\frac{1}{x},\phi=\frac{1}{z}-\frac{1}{x}$，对函数$\phi=\phi(u,v)$，求证$\frac{\partial\phi}{\partial u}=0$
   出现的错误：（分析错误）ϕ是$x,z(x,y)$的函数，即是x,y的函数，而x,y可以由u,v分别表示，所以可以构造出树形图，则
   $$\begin{aligned}\frac{\partial\phi}{\partial u}&=\frac{\partial\phi}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial\phi}{\partial y}\frac{\partial y}{\partial u}\\&=(-\frac{1}{z^2}\frac{\partial z}{\partial x}+\frac{1}{x^2})+(-\frac{1}{z^2}\frac{\partial z}{\partial y})\frac{\partial y}{\partial u}\end{aligned}$$
   而使用$u=x,v=\frac{1}{y}-\frac{1}{x}$解出$y=\frac{u}{uv+1}$，带入得
   $$\frac{\partial\phi}{\partial u}=-\frac{1}{z^2}\frac{1}{x^2}(x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y})+\frac{1}{x^2}=0$$